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AEROSPACE SYSTEMS and MISSION ANALYSIS RESEARCH

Status Report for the Period

1 July through 30 September 1967

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I. INTRODUCTION

During the summer research of the ASMAR Program continued as previously outlined under the sponsorship of NASA/OSSA Launch Vehicle and Propulsion Program Division (NASA Contract NASr-231).

Mr. J. P. Layton, the research leader, has left for California where he will spend the year at the Lawrence Radiation Laboratory. He remains in continuous touch with the work going on in Princeton, however.

During the coming academic year the research will be supervised by Professor P. M. Lion who also will have direct responsibility for the work in Trajectory Analysis. Dr. Robert Vichnevetsky will direct the Systems Analysis Group in the absence of Mr. Layton. Mission Analysis Research will be coordinated between Professor Lion and Dr. M. Handelsman, visiting senior research scientist.

Three new graduate students have joined the group this month: John Elliot (BS, Cornell), Alain Kornhauser (MS, Penn State), and Paul Van Woerkom (BS, Delft Tech. University, Netherlands). Together with the two remaining students, Michael Minkoff and George Hazelrigg, we now have a total of five. Mr. Jean Peltier has returned to France after submitting his Master's thesis. Undergraduates involved in this program are Mr. Charles Kalmbach ('68) and Mr. Ronald Chin ('69).

Our programming staff now includes Mr. John Campbell, who has been largely responsible for bringing the programs to the state of readiness that we have today. He is assisted by Mrs. Alexandra Shulzycki. Miss Nancy Payne provided considerable assistance during the summer, particularly in converting our programs to IBM System 360; she has left to pursue graduate study at Northeastern University.

Other members of our staff are Miss Frances Allison, who handles all administrative and secretarial matters and Mr. Leonard Jas, draftsman. Also, Dr. Michael Mintz, consultant, has made a substantial contribution to the work going on under Dr. Vichnevetsky.

Looking back it now appears clear that 1966-67 was a year of transition, changing from old programs to new programs, and involving a considerable change of personnel. The group has now shaken down, the basic programs have been developed, and we look forward to 1967-68 with enthusiasm.

II. SPACEFLIGHT TRAJECTORY ANALYSIS RESEARCH

A. Program Development

The primary effort during this period has been directed toward the completion of two programs, Lion 1 (John Campbell) and the Multi-impulse Program (Michael Minkoff).

The program Lion 1 is essentially completed. It is converging successfully and has duplicated results of similar programs. The major requirement now is the exercise of this program over the range of useful parameters. Unfortunately, this exercise is somewhat hampered due to the new policy of direct computer charging. In the meantime, documentation of the program has been started. Improvements to the program include the following:

1. A subroutine allowing the selection of the launch vehicles has been included. The following vehicles can be chosen:

    1. Atlas (SLV3C)/Centaur
    2. Saturn IB/Centaur
    3. Titan III C (1207)
    4. Titan III C (1207)/Centaur

These vehicles are represented by curves of initial mass vs heliocentric injection energy, as provided by JPL.

2. A study to match the precision requirements of the program to the IBM 360 computer has been completed. A saving in execution time of about 50% has been obtained by the selective use of single precision without any detrimental effects on the program. Conversion to the CDC 6600 is in progress.

The Multi-impulse Program is also essentially complete. During the summer Mr. Minkoff modified the optimal multi-impulse program for use on the IBM 7094 at Ames Research Center. Initially, modifications in the numerical procedures for obtainin solutions of Lambert's problem and the state-transition matrix were made. The modifications, based on suggestions of Mr. S. Pines of Analytical Mechanics Associates led to slightly faster computational times. Major improvements in speed of the program were obtained by going from the gradient and Partan methods to a conjugate gradient method. The particular procedure used was based on a Fletcher-Powell variation of Davidon's method. The improved convergence resulted in the computation of optimal three-impulse trajectories in one-fourth to one-tenth of the previous computer time. The program was tested for a variety of departure and arrival orbits in two and three dimensions. Optimal plane-change trajectories for near-Hohmann transfers were obtained in approximately 10 seconds.

An extension of the program, suggested by Mr. A. Mascy at Ames, allows the procedures to be used for optimizing Δv for departure from and arrival to circular planetary orbits. Also, gravity loss approximations can be provided.

The program is currently being used by the Mission Analysis Division of OART at Ames Research Center and will be available on the Princeton IBM 360/50 shortly.

B. Analytical Results

Professor Lion's paper on "Sufficient Conditions for Optimal Fixed-Time Impulsive Trajectories" was presented at the XVIII IAF Congress in Belgrade, Yugoslavia in September. Using these results, a quick check can be made to ensure the optimality of trajectories which satisfy the necessary conditions (e.g., those generated by the Multi-impulse Programs). Assuming an inverse square field, this check can be made using only algebraic operations; no integration is required. An attempt is being made to extend this result to finite thrust trajectories.

Mr. Jean Peltier has submitted the manuscript of his Master's thesis. The thesis is a study of two and three impulse trajectories with and without terminal coasts between circular coplanar orbits. The principal result is a map of the $\emptyset$-T plane showing the areas where three impulse trajectories represent an improvement over 2-impulse trajectories. Many qualitative properties of this map have been derived mathematically by Mr. Peltier. The thesis is now being reviewed and will be issued shortly as a report.

III. AEROSPACE SYSTEMS ANALYSIS

A. General

Work on aerospace systems analysis is receiving increased attention both in the ongoing efforts and in our planning. In addition to the analysis of nuclear rocket propulsion systems directed by Dr. R. Vichnevetsky as described below and in several ASAR Memos (See list in APPENDIX A), Dr. Handelsman has been carrying out the analysis of space communications systems, in particular radar subsystems for the survey of planetary surfaces (ASAR Memo No. 3) and asteroid detection (ASAR Memo No. 5). Work on space navigation, guidance and control systems will be initiated during the Fall Term under the direction of Professors Graham and Lion.

Dr. Vichnevetsky's Advanced Methods of Systems Analysis Seminar notes have been assembled and a limited number of copies are available to workers in this topic area. In addition to employing these methods in our nuclear rocket system analysis, we are attempting to identify those methods that are broadly applicable for the sytems described above and others.

B. Nuclear Rocket System Analysis

Systems analysis work under Dr. Vichnevetsky's direction has emphasized the completion of a mathematical model for nuclear rocket systems, initially for the analysis of attainable performance.

The mathematical model is aimed at its further implementation as a computer program. This program is partially completed.

The computer program will be used specifically for the derivation of the interrelation between the internal engine parameters and the external system performance parameters.

The internal engine parameters are those which are either constrained by technology, or should be investigated over the adjacent ranges in a search for optimal systems. The most important internal engine parameters are: maximum material temperature, chamber pressure, nozzle expansion ratio, and reflector thickness.

The external engine performance parameters are those which characterize an engine from the mission analysis standpoint. There are basically three external engine performance parameters: engine mass M_e , effective jet velocity V_j , and thrust F . Thrusting time is also an important consideration. In addition to establishing direct relationships between internal and external engine parameters, the computer program is being organized so as to derive the sensitivity functions (or partial derivatives) of the external with respect to the internal parameters. An optimization algorithm is being used to obtain, for each set of specified thrust and effective jet velocity, the engine with minimum mass.

For a given reactor size the nuclear criticality condition can always be satisfied by a proper uranium loading. Therefore, the thermodynamic part of the mathematical model can be separated from the nuclear equations. In the program being programmed, a simple relationship between reflector thickness and core radius is being utilized. A program, however, is being rewritten for the analysis of mass savings achievable by varying the core reflector geometry. This program is based on a 2-program neutron diffusion theory.

The model described in the preceeding section will be used as an input to a mission-oriented analysis of propulsion parameters, based on the multi-level optimization method described in ASAR Memo No. 4.

Preliminary analysis of other propulsion systems for space missions has been started. The ultimate evaluation of these propulsion systems will also be based on multi-level mission oriented optimization to allow a comparison of potential performance with respect to the nuclear rocket presently being analyzed in detail.

IV. INTERPLANETARY-PLANETARY MISSION ANALYSIS RESEARCH

1. During the summer, an interesting comparison of the capabilities of three launch vehicles was made using Gordon II: Atlas (SLV3C)/Centaur, Titan IIIC(1207)/Centaur and Saturn 1B/Centaur. The results are shown in Table I. The mission is a 600 day, solar electric, Jupiter flyby, assuming coplanar circular orbits. The trajectories are fully optimized: thrusting program, V_j, FMO, power, and C_3. Transfer angle has also been optimized. The resulting net masses are:

Atlas (SLV3C)/Centaur	174.86 kg
Titan IIIC(1207)/Centaur	1943.39 kg
Saturn 1B/Centaur	1778.50 kg

The specific mass of the powerplant was taken to be α = .026 kg/w . Tankage and structure factor were assumed to be .06 and .08, respectively.

Table 1

	A/C	T/C	S/C
Initial Mass (kg)	696.21	3563.62	3485.39
Powerplant (kg)	303.32	913.30	972.72
Fuel (kg)	153.15	397.96	429.52
Fuel Tanks (kg)	9.19	23.88	25.77
Structure (kg)	55.70	285.09	278.83
Net Mass (kg)	174.86	1943.39	1778.50
Initial Power (kw)	11.67	35.13	37.41
Efficiency (η)	.54	.55	.54
C_3 (km/sec)^2	16.20	47.86	43.86
V_j (km/sec)	32.532	32.836	32.805
FMO (m/sec^2)	.5583(10^{-3})	.3272(10^{-3})	.3565(10^{-3})

2. Dr, Handelsman has completed a preliminary study of the feasibility of an on-board microwave radar for detection, range and position-angle measurement of asteroids from a spacecraft moving within or through the belt. This capability is needed to explore the belt,[1,2] or to furnish input data for an asteroid collision-avoidance system.[3,4,5] For exploration, this radar would be supplemented by additional equipment for detailed measurements of asteroid size, orbital elements, rotation or spin, and surface characteristics (photography, reflectivity, dielectric constant, roughness, etc.). For collision avoidance, additional subsystems would be required for target tracking, trajectory prediction and maneuverability.

Sample trajectories have been calculated using three boosters to launch a solar-electric propelled spacecraft into heliocentric orbit: (1) Atlas (SLV3C)/Centaur, (2) Titan IIIC(1207)/Centaur, and (3) Saturn 1B/ Centaur. The electric power is used for propulsion and for electronic needs such as radar and communication. Suitable nuclear-electric power systems could be used instead of solar-electric; these are not investigated in this report. The trajectories are optimal in the sense of maximum payload, open-angle, fixed-time, two dimensional (circular orbits) with hyperbolic excess velocity (C_3), jet exhaust velocity and initial electric power optimized for maximum payload. The electric powerplant specific mass factor (α) is 0.026 kg/watt, and tankage and structure factors are 0.06 and 0.08, respectively. The electric engine efficiency (η) as a function of I_{sp}, the power fall-off law with solar distance, and the booster characteristics are as prescribed by JPL. For further details concerning the trajectory analysis, see Ref. 6. The trajectories include rendezvous (match to local heliocentric satellite velocity) at 2 A.U (400-400 days), 3 A.U. (700 days), 3.5 A.U. (800-900 days) and 4 A.U.

(900-1000 days), Jupiter flybys (600 days), a special trajectory which is flown optimally to 2 A.U. and then coasts through the belt, and another trajectory which has its aphelion at 4.5 A.U. and passes through the belt on outbound and inbound legs. Summarizing the trajectory results, boosters (2) and (3) can furnish adequate payloads (1000 to 2000 kg) and power levels (30 to 130 kw initial), while booster (1) is not adequate. For further details see Ref. 7.

An asteroid distribution model is first derived based upon Marshall's[4] flux. Assuming spherical particles, an average density, and a uniform volumetric distribution within the belt, it is shown for particles of mass > 1 gm that

$$N \approx 10^{16}\, d^{-2.31} \tag{1}$$

N is the total number of particles of diameter d (meters) or greater in the belt.

Next, using a standard pulsed radar, it is shown that

$$\hat{P} = \frac{(4)^4 \lambda^2 k T_e B R^4 X_1}{\pi^2 \eta^2 \sigma_o d^2 D^4 (N_p)^x} \tag{2}$$

where $\hat{P}$ = peak power, λ = wavelength, k = Boltzmann's constant, T_e = system noise temperature, B = system bandwidth, R = detection range, X_1 = signal-to-noise ratio (per pulse), η = antenna efficiency, σ_o = ratio of radar echo area to target cross-section, D = antenna diameter, and N_p = number of pulses integrated per range bin per decision. The average power $\overline{P}$ is $\hat{P}$ multiplied by τ (pulse length) and by f_R (pulse repetition rate). Using reasonable values such as $B \approx 4 \times 10^5$ cps to accommodate the maximum expected doppler shift

($\approx 3 \times 10^5$ cps) which is determined by λ (S band; λ = 0.131 meters) and the maximum asteroid radial velocity relative to the spacecraft ($\approx$ 20,000 m/s) obtained from the trajectories, $T_e = 440\ ^oK$, $X_1 = 25$, $\eta = 0.7$ and $\sigma_o = 0.1$, there obtains

$$R \approx 10^3 \cdot D \cdot (d)^{1/2} \cdot (\hat{P})^{1/4} \cdot (N_p)^{x/4} \tag{3}$$

Two radars representative of lower and upper bounds of radar size are then examined: (1) D = 3m , $\hat{P} = 10^4$ watts , and $\bar{P}$ = 100 watts and (2) D = 6 m , $\hat{P} = 10^6$ watts and $\bar{P}$ = 10 kilowatts . This is based upon $\tau = 10 \times 10^{-6}$ sec. and $f_R = 10^3$ cps. The latter corresponds to a maximum unambiguous range (for fixed f_R) of 150 km. This fixes D , $\hat{P}$ and N_p in Eq. (3).

Three types of beam scanning modes are next examined: (A) Fixed beam parallel to flight path, (B) Fixed beam transverse to flight path and (C) Beam scanned in a plane transverse to flight path. Scan C would require either a phased array, or a spinning spacecraft, or a mechanical-scanning antenna system. As an example of the results, for a length of trajectory within belt $\approx$ 3.3 A.U., and for scan mode (B), the coverage volume of radars (1) and (2) are shown[7] to be $V_i \approx k_i d$ (cu.m.), where $k_1 \approx 5 \times 10^{19}$ and $k_2 = 10^{21}$, based upon $N_p \approx 40$ and $x \approx 0.75$.

Combining the asteroid distribution with the radar results, it is then shown that, for scan mode (B), the cumulative number of detections (hits H) of asteroids of diameter d or greater is given by

$$\text{radar 1:} \qquad H \approx 5d^{-1.31} \tag{4}$$

$$\text{radar 2:} \qquad H \approx 100d^{-1.31} \tag{5}$$

The inverse power dependence of H upon d results from the fact that while radar coverage volume increases as d , the volumetric concentration of particles decreases more rapidly as $1/d^{2.31}$. Hence, more detections of smaller sizes at close-up ranges occur, or fewer detections of larger asteroids, despite their longer ranges of detection.

Curves of radar range R , fractional volume f (of entire belt) searched by the radar, and cumulative number of hits per traverse H , for scan mode (B), and number N of asteroids of diameter d or greater vs asteroid diameter d are shown in Figure 1. These curves do not apply for $d < 0.01$ meters, where $m < 1$ gm and the flux distribution Eq. (1) does not apply. In addition, for $d < 0.04$ m , Raleigh scattering takes over and the radar echo area of aspherical target decreases with d . This causes the curves for R , f and H , for $d < 0.04$ m , to bend downward from the straight lines shown. The curves show that the number of detections of sizable objects ($d \geqq 0.1$ meters) is surprisingly few. Despite their vast number, as given by the assumed distribution, the asteroids are spread over an enormous volume. For $d \gtrsim 0.1$ meters, radar 2 can furnish initial acquisition warning times of between 6 to 24 seconds, depending upon the trajectory to a collision-avoidance system. For much smaller sizes, the problem requires further investigation.

The results of Figure 1 are preliminary; no attempt has been made to optimize key radar parameters such as λ , D , τ , scan mode, or the overall power supply-trajectory-radar system. In addition, it may be worthwhile to investigate the bistatic radar mode (transmitter on earth, receiver in spacecraft) and the use of optical (laser) rather than microwave bands.

References

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2. Friedlander, A. L. and Vickers, R. S., "A Survey of Missions to the Asteroids," IITRI Report #M3.

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6. "Solar-Electric Space Mission Analysis," Progress Report Jan.-Mar., 1967, ASMAR Group, Department of Aerospace and Mechanical Sciences, Princeton University.

7. Handelsman, M., "Preliminary Study of Radar Subsystem for Asteroid Belt Exploration Spacecraft," ASAR Memo No. 5, ASMAR Group, Department of Aerospace and Mechanical Sciences, Princeton University, October 3, 1967.

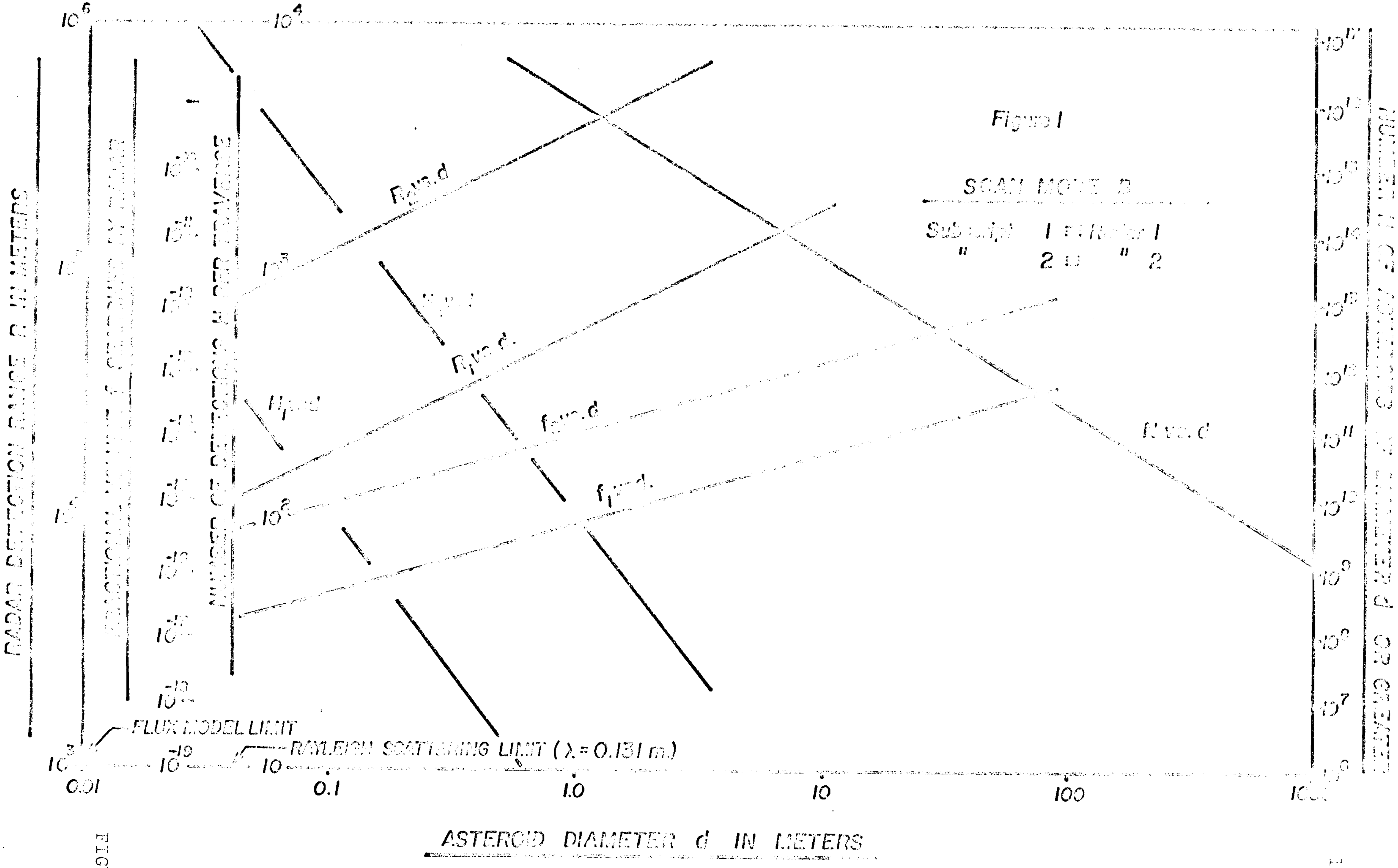
RADAR DETECTION RANGE R IN METERS
NUMBER OF DETECTIONS N PER TRAVERSE
NUMBER N OF ASTEROIDS OF DIAMETER d OR GREATER
Figure I
SCAN MODE B
FLUX MODEL LIMIT
RAYLEIGH SCATTERING LIMIT (λ = 0.131 m.)
0.01
0.1
1.0
10
100
ASTEROID DIAMETER d IN METERS

FIGURE 1

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AEROSPACE SYSTEMS and MISSION ANALYSIS RESEARCH (ASMAR) PROGRAM

1 October 1967

List of ASAR Memos

ASAR Memo #1, Record of Meeting on Nuclear Rocket System Analysis on February 8, 1967, R. Vichnevetsky, 24, February 1967.

ASAR Memo #2, Objectives and Progress Report - Nuclear Rocket Systems Analysis, R. Vichnevetsky, 23 May 1967.

ASAR Memo #3, Introduction to the Theory of Synthetic Array Radar, M. Handelsman, 12 July 1967.

ASAR Memo #4, Propulsion System Optimal Design by Multi-Level Optimization, R. Vichnevetsky, 15 July 1967.

ASAR Memo #5, Preliminary Study of Radar Subsystem for Asteroid Belt Exploration Spacecraft, M. Handelsman (in preparation).